Network Theory and Military C2 Systems

Where Axioms and Action Meet

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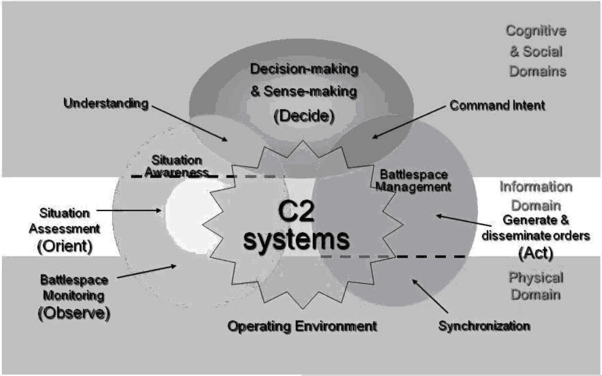
Operational setting

Since the end of the Cold War the character of military operations has changed. Operations have become expeditionary in nature, combining defence, diplomacy and development. Coalitions include non-military organizations, host nation police forces, international organizations, commercial suppliers, etc. All these partners, and their characteristic way of operating, must find a place within the overall C2 structure.

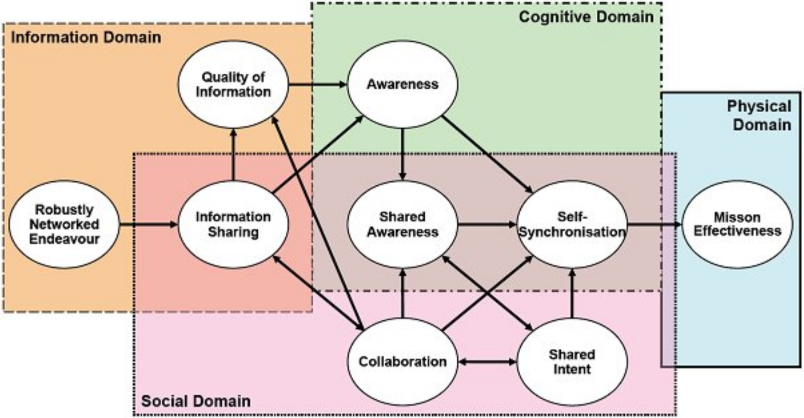
The goal is to share information by linking the disparate (command) and control systems, and coordinate action.

C2 modelled in four domains

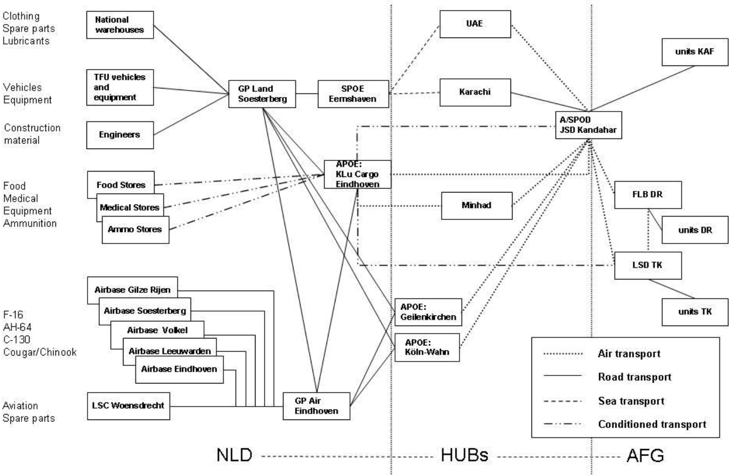
Military action is constrained by norms, policies, standard operating procedures, rules of engagement, and the like. This shows that military C2 has socio-organizational, cognitive and technological aspects.



Quantifying the benefits of networked operation



Logistics network of NLD-forces in Afghanistan (AFG)



Research issue

Multi-layer networks pose new questions and reasoning about multiple networks that are linked at various points. Understanding the incentives of the various actors can be used to train and manage the actions of the actors/partners, so that various parts of the physical, information, social and cognitive domain obtain desirable properties. This agile federation of components of the operational environment may be used to deepen our understanding of successfull military operations.

We need to reveal the *interplay* between aggregate qualities found in the *military C2* literature and the *network approach* that uses metrics and statistics. This network approach also has to be extended to multi-layer networks.

Merging two approaches

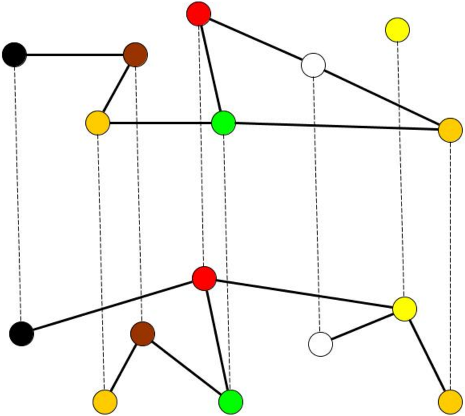
We merge the network approach (with various network statistics) with the C2 approach (with soci-technical metrics). Merging proceeds as follows:

- Networks are multi-layered networks that influence each other.
- Each actor is assigned an objective function, incorporating endogenous network statistics and exogenous covariates.

This function can be used to evaluate how network statistics and covariates influence the network dynamics and, in turn, are influences by the same dynamics. It also shows how the C2 structure adapts itself to a changing environment.

Two layers

A node typically is not just part of one type of network, but simultaneously belongs to multiple networks.



Layer 1: Information network

Networked operations offer decisive advantage through the timely provision and exploitation of (*feedback*) information and intelligence to enable effective decision-making and agile actions.

We focus on the aspect of information sharing in collaboration networks and discuss a feedback model for situational awareness, that combines exogenously given characteristics of nodes with their positioning within the information network topology.

Layer 1: Information feedback

We let A be the adjacency matrix, where $a_{ij} \in [0, 1]$ is the extent to which value from node j is *usable* or *transferable* to node i regarding the improvement of i's situational awareness. We take $a_{ii} = 0$ for each node i.

Layer 1: Feedback of operational links

We combine the transferred situational awareness, which depends on the network, with exogeneous values.

- *Combining operational feedback links with exogenous value.* Given a scalar α and a vector of exogenous characteristics b, the value v is the unique solution of the equation:

  $$v = \alpha A v + b.$$

  We say that the situational awareness v is 'confirmed' by the network structure and b.
- The surplus $v - b$ may be ascribed to the network topology.

Layer 1: Iteration of updating of situational awareness

Updating information in m steps yields the situation awareness v_m, which for $m \geq 1$ is defined recursively as follows:

$$v_0 = b; \quad v_1 = b + \alpha A v_0 \quad \dots \quad v_m = b + \alpha A v_{m-1}$$

Taking the limit of m to infinity, we get

$$v = \lim_{m \to \infty} v_m = \lim_{m \to \infty} \sum_{k=0}^{m} \alpha^k A^k b = (I - \alpha A)^{-1} b$$

By iteration, nodes also receive information from nodes which are not adjacent, but are two, three, or more steps away.

Layer 1: Relation to principal eigenvalue

The informational value/situational awareness derived with this model that includes exogenous characteristics, is similar to the Perron eigenvector c in case we let α approach $1/\lambda$ from below.

Theorem (Janssen & Monsuur, 2010, Collective decision making)

Suppose that A is primitive. Then

$$\lim_{\alpha \uparrow 1/\lambda} (1 - \alpha\lambda)v = (d^T b)c.$$

Here, d is the (positive) eigenvector of A^T corresponding to λ with $d^T c = 1$.

Layer 2: Social network

What we did for the information network, can also be done for social networks.

$$w = \gamma C w + d.$$

Of course, one may also consider metrics for centrality, clustering, domination, etc.

Merging layer 1 and 2

To give an example, how two networks influence each other, consider the following merging procedure. Instead of only taking into account exogeneous values b for situational awareness v in the expression $v = \alpha A v + b$, we also include the exogenously given value w derived from acting in the social network:

$$v = \alpha A v + b + \sigma w$$

and, in turn, for determining the social position, we include its information value v derived from its acting in the information network:

$$w = \gamma C w + d + \tau v,$$

where $\sigma, \tau \leq 1$.

The procedure to determine a next state of the system (1)

Let the information network be represented by the matrix A, while the social network is represented by the matrix C. The stochastic actor based approach consists of a few steps:

- Choose an agent i.
- Determine v_i and w_i from

$$v = \alpha A v + b + \sigma w$$

$$w = \gamma C w + d + \tau v$$

  and determine the overall utility for agent i of the structure (A, C) using the objective function $f_i(A, C)$. For example $f_i(A, C) = 2/3 v_i + 1/3 w_i$.

The procedure to determine a next state of the system (2)

- Evaluate all possible changes in A and C that agent i is able and allowed to realize. Compute the new values $v_{new,i}$ and $w'_{new,i}$ from

$$v_{new} = \alpha A' v_{new} + b + \sigma w_{new}$$

$$w_{new} = \gamma C' w_{new} + d + \tau v_{new}$$

  and determine, for each of these possible changes the objective function value $f_i(A', C')$ of the structure (A', C').

- Use of a specific probabilistic mechanism to determine the actual choice of agent i. A higher value $f_i(A', C')$ means a higher probability of going to the state (A', C').

Extensions of the objective function: Behavioral constraints

The objective function of an agent is a linear combination of several effects. These effects may include situational awareness, control (centrality), norms and behavioural effects like the restriction that, due to security regulations, only NATO partners may be selected for new links.

Extensions of the objective function: total networked effect

In the literature it is assumed that just one specific arrangement of links and nodes creates value. These arrangements are sub-networks that form a cycle. In a cycle, the functions of nodes flow into each other over a path that revisits at least one node once. To give an indication of the magnitude of networked effects, it is suggested to use the largest eigenvalue of the adjacency matrix, which is a measure of the multiplicity of internal paths:

$$\lambda = \max_{x>0} \min_{i} \frac{(Ax)_i}{x_i}.$$

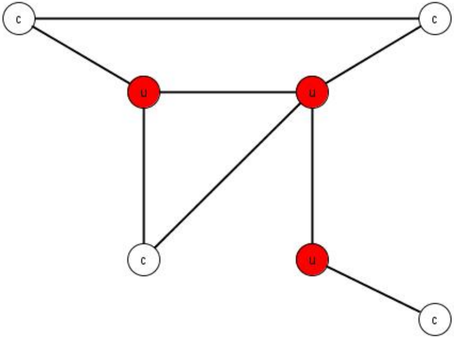
Extensions of the objective function: Game-theoretic metric for control

Let the cooperative game (V, ν) be defined by letting $\nu(F)$ be the number of unordered pairs $x, y \in F$ such that there is at least one path from x to y that is contained entirely in F, where F is any subset of V. For a node v_i, the Shapley value or Shapley control is defined as the expected marginal contribution of v_i to every subset F that contains v_i:

$$Sh(\nu)(v_i) = \sum_{F \subset V, v_i \in F} \frac{(f-1)!(n-f)!}{n!} (\nu(F) - \nu(F \setminus v_i)).$$

Extensions of the objective function: Uncovered set

We let U or $U(G)$ be the uncovered set: $U = \{v \in V\colon$ there is no node $w \in V$ that covers v in the network $G\}$.



Investigating emergent network properties and C2 qualities

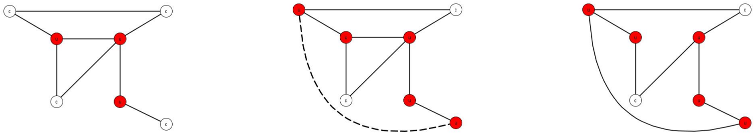
Given some objective function, we try to determine the emergent properties of the network that results by iterating the stochastic actor based procedure. For example, given

$$f(A, C) = \alpha v + \beta w + \gamma\lambda(A) + \delta Sh(\nu, C) + \epsilon Uncov,$$

what combination of coefficients $\alpha, ..., \epsilon$ gives desirable properties for the various parts of the physical, information, social and cognitive network that emerges?

We present results with respect to one effect: the uncovered set.

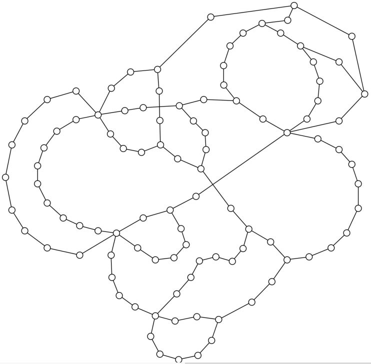
Dynamics: Escaping the subdued position of being covered



Emerging networks

Theorem (Janssen & Monsuur, 2011)

If we simulate the evolution of a network using the notion of covering, then, starting with all nodes uncovered, minimal network structures have a bubble like structure.

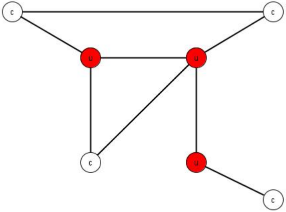


Axioms

A *center* ϕ assigns to any network $G = (V, E)$ a non-empty subset $\phi(G) \subset V$. The center ϕ_{uc} assigns to a network G the set of uncovered nodes.

We consider the following axiom for a center ϕ:

- A center ϕ has the *mediator property* if for each pair of distinct nodes a and b, there is a shortest path connecting these nodes, such that any node in between a and b on this path is in $\phi(G)$.



Characterisation of uncovered set

Theorem (Monsuur & Storcken, 2004, Operations Research)

The center set ϕ_{uc} is the only inclusion minimal set of nodes that is compatible with structural equivalence, has the mediator property and is stable.

Future work

- Investigate the interaction between various effects
- Investigate the network statistics of the emerging networks and their relation to C2 qualities
- Employ OR techniques to simulate military coalitions as multi-layer networks of atomic actors
- Employ intelligent agent technology, with the actors being represented by software agents with an internal structure derived from the OODA model

Presentation based on

- H. Monsuur, T.J. Grant and R.H.P. Janssen (2011) *Network Topology of Military Command and Control Systems: Where axioms and action meet*. In: Computer Science , Technology and Applications, Vol 3: 1-27